

$$1) a + b\sqrt{2} \quad ; \quad a, b \in \mathbb{Q}$$

a) asociativni:

$$\begin{aligned} L &= [(a + b\sqrt{2}) \cdot (c + d\sqrt{2})] \cdot (e + f\sqrt{2}) = [ac + ad\sqrt{2} + bc\sqrt{2} + bd\sqrt{2}] (e + f\sqrt{2}) = \\ &= ace + ade\sqrt{2} + bce\sqrt{2} + 2bde + acf\sqrt{2} + 2adf + 2bcf + 2bdf\sqrt{2} \end{aligned}$$

$$\begin{aligned} P &= (a + b\sqrt{2}) \cdot [(c + d\sqrt{2}) \cdot (e + f\sqrt{2})] = (a + b\sqrt{2}) \cdot [ce + cf\sqrt{2} + de\sqrt{2} + 2df] = \\ &= ace + acf\sqrt{2} + ade\sqrt{2} + 2adf + bce\sqrt{2} + 2bcf + 2bde + 2dfb\sqrt{2} \end{aligned}$$

$$L = P \Rightarrow \text{Ok}$$

b) neutralni prvek:

$$(a + b\sqrt{2}) \cdot e = a + b\sqrt{2}$$

$$e = 1$$

c) inverzni prvek:

$$(a + b\sqrt{2}) \cdot a^{-1} = e = 1$$

$$a^{-1} = \frac{1}{a + b\sqrt{2}} \rightarrow \begin{aligned} a + b\sqrt{2} &\neq 0 \\ a &\neq -b\sqrt{2} \end{aligned}$$

$$\frac{a}{b} + \sqrt{2}$$

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$$1) a \oplus b = a \oplus b + ab$$

20

$$\text{asociativitate: } (a \oplus b) \oplus c = a \oplus (b \oplus c)$$

$$(a + b + ab) \oplus c = a + b + ab + c + ac + bc + abc$$

$$a \oplus (b + c + cb) = a + b + c + cb + ab + ac + abc$$

$$\Rightarrow 0_K$$

$$\text{neutru' element: } a \oplus e = a$$

$$a + e + ae = a$$

$$e \cdot (1+a) = 0$$

$$e = 0$$

$$\text{inversul element: } a \cdot a^{-1} = e$$

$$a + a^{-1} + a \cdot a^{-1} = e$$

$$a^{-1}(1+a) = e - a$$

$$a^{-1} = \frac{e-a}{1+a} \Rightarrow e=0$$

$$\underline{a^{-1} = \frac{-a}{1+a}}, \quad a \neq -1$$

$$M = \mathbb{R} \setminus \{-1\}$$