Comparison of linear, cubic spline and akima interpolation methods

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August 30, 2007

1 Introduction

In the correlator software the delay times are calculated in advance with a coarser intervals than the intervals actually used. Within the correlator software the calculated coarser delay time data is used to interpolate the actual time intervals by applying a parabolic spline. In this report the effect of other types of interpolation methods namely linear, cubic spline and akima method are investigated.

A cubic spline curve is a piecewise cubic curve with continuous second derivative. The resulting curve is piecewise cubic on each interval, with matching first and second derivatives at the supplied data-points. The second derivative is chosen to be zero at the first and last points[1].

The akima method is also based on piecewise polynomials, but differs from the spline by the conditions imposed at the data points. In his method, the interpolation function is a cubic polynomial the coefficients of which are fixed, between every pair of successive data points, by the condition that the function passes through the points with specified slopes. These slopes are determined by a "local" procedure, the slope at a given datum point being a weighted average of the slopes of the line segments connecting that point with those on either side[2].

2 Analysis

The interpolation methods are implemented using the *GNU Scientific Library (GSL)*[3] functions. All three methods are first applied to two generic cases in order to verify the implementation and shown in figure (1). The left hand side of figure (1) shows the methods applied to a randomly chosen sample points while the right hand side shows methods applied to a sine function. The sample points to represent the Sine function is also chosen randomly.

In order to compare all three interpolation methods they are applied to a real delay table data. A reference delay table data is computed for 0.001 second intervals and the sample points are chosen out of the delay table data with an interval of 10, 1 and 0.1 seconds respectively. The interpolation methods are applied to each of these sample data and the results are computed at each point intersecting the reference delay table data. In this way it is possible to compare the interpolated data with the reference delay table data (see figure (2)).

The difference between the reference delay table and interpolation computed by akima method is shown in figure (3). The difference is calculated at each reference data points by using sampling intervals of 0.1 second for one sub-scan data. As it can be seen from this figure the difference between the akima method and the reference data is about 10^{-16}.

The maximum error for each interpolation methods and for each sample time intervals are calculated and presented on the left hand side of figure (4). Here again the calculation is performed for only one sub-scan data. The maximum error is presented on a logaritmic scale. The maximum error with the akima method is much lower than the maximum error with the linear and cubic spline interpolation methods. Also the maximum error obtained by the akima method using sampling intervals of 10 second is about the

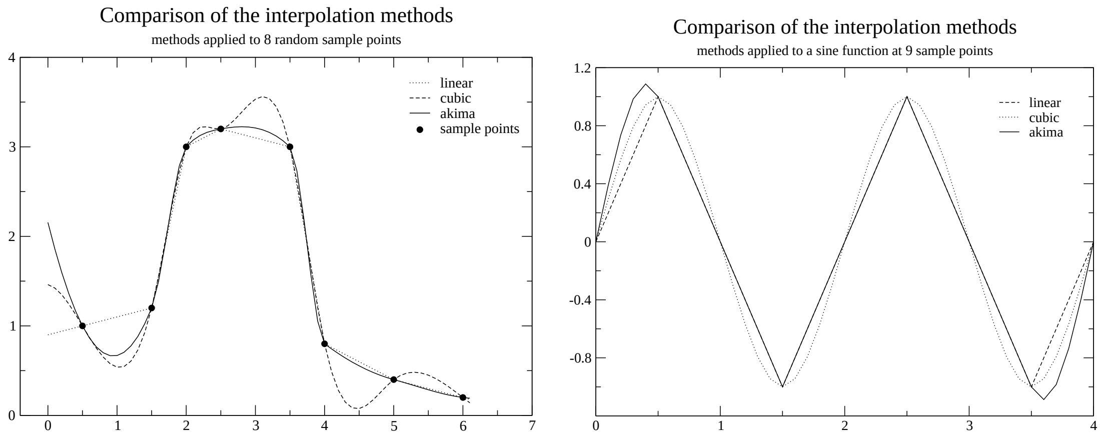

Figure 1: *Comparison of the interpolation methods applied to a Sine function using 9 random data points (left) and applied to any 8 random data points (rigth).*

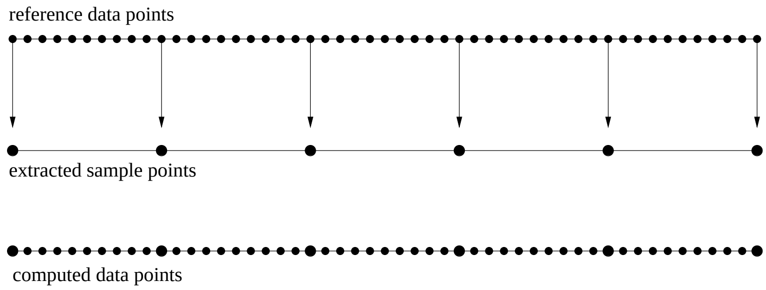

Figure 2: *Scetch of reference data points (top), extracted sample points (middle) and computed data points (bottom).*

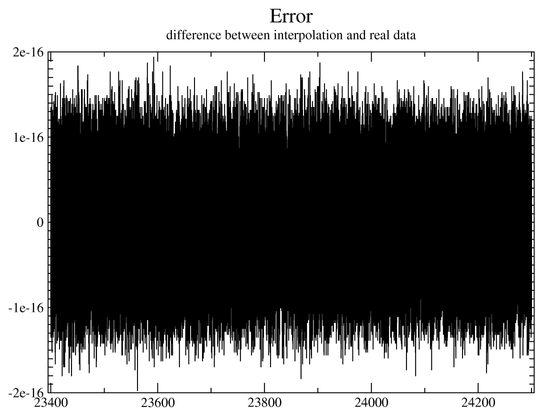

Figure 3: *The difference between the reference delay table and interpolation computed by akima method with sampling intervals of 0.1 second for one sub-scan data.*

same order with the maximum error obtained by the linear and cubic spline interpolation methods using sampling intervals of 0.1 second. Another interesting result is that when the error curves are compared the akima method has a converging behaviour with decreasing sampling intervals but this converging behaviour is not evident in other two interpolation methods.

The right hand side of figure (4) shows the maximum error obtained for all sub-scans (46 sub-scans in total)with akima method. The figure shows that the difference between the maximum errors for various sub-scans are very small compared to the maximum errors.

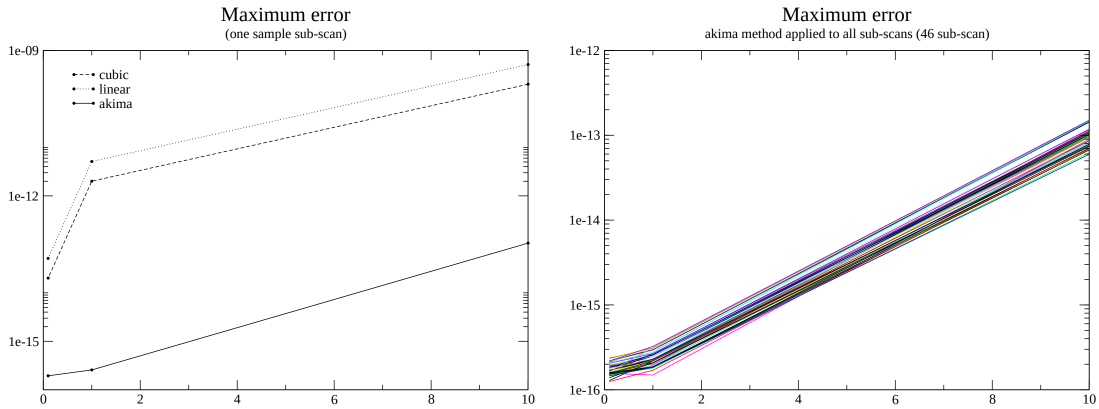

Figure 4: *A comparison of maximum errors obtained for each interpolation methods for one sub-scan data (left) and maximum errors obtained for akima method for all sub-scan data (right).*

3 Conclusions

Three interpolation methods are compared namely, linear, cubic spline and akima method, by the maximum errors obtained from each method applied to a sample data. It is shown that the maximum error obtained by the akima method is much lower than the linear interpolation and cubic spline methods. As a result of this study the akima methos is implemented to the correlator software in place of parabolic interpolation used in the software. The preliminary results show that the implemented part of the routine runs about 1.5 times faster than the original routine. The use of akima method will also allow to use coarser time intervals to calculate the delay table.

References

[1] McKinley, S., Levine, M., *Cubic Spline Interpolation*, Math 45: Linear Algebra, http://online.redwoods.cc.ca.us/instruct/darnold/laproj/Fall98/SkyMeg/Proj.PDF

[2] Akima, H., 1969, *A Method of Smooth Curve Fitting*, ESSA Tech. Rep. ERL 101-ITS 73 (Washington, D.C.: U.S. Government Printing Office)

[3] *GNU Scientific Library*, http://www.gnu.org/software/gsl/manual/index.html